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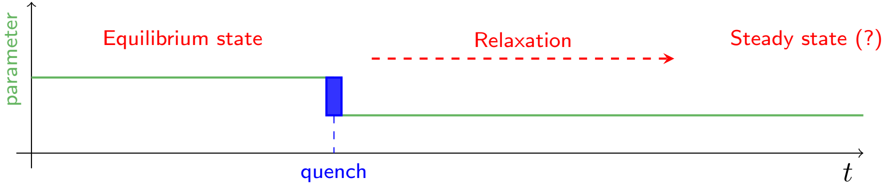
# Transport properties as a tool to study quench-induced dynamics in 1D systems

Alessio Calzona

QTC Espoo - August 7, 2017

**Definition**: a change in time of the parameter(s) that govern the dynamics of an isolated quantum system (i.e. under unitary time-evolution).




Convenient way to bring a system **out-of-equilibrium** and to study its following **relaxation** toward a steady state (?).

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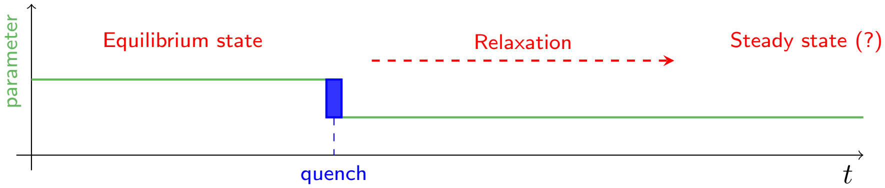


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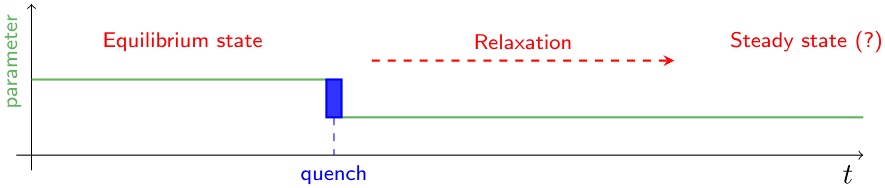


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- Does the steady state exist? If yes, which kind of state is it?
- Which are the features of the relaxation dynamics?

# INTEGRABLE SYSTEMS

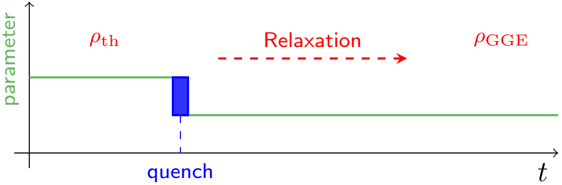
Low energy physics of 1D interacting fermion systems → **Luttinger liquid model** (LL)

**Integrable system**:

- complete set of conserved quantities;
- a steady state is reached (in the thermodynamical limit) but it retains a **strong memory** of the initial state;
- local observables relax to **non-thermal values** described by the GGE density matrix

$$\rho_{\mathrm{GGE}} = \frac{e^{-\sum_m \lambda_m I_m}}{Z_{\mathrm{GGE}}}$$

Rigol et al. PRL (2007)



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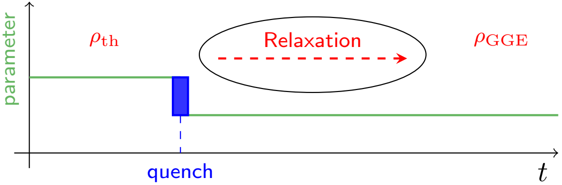
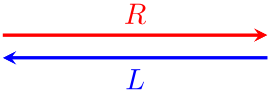
Our goal: key features of the relaxation process towards GGE in LL



# LUTTINGER LIQUID




$$\hat{\mathcal{H}}_0(x) = iv_{\mathrm{F}} \left( \hat{\psi}^\dagger_R(x)\partial_x\hat{\psi}_R(x) - \hat{\psi}^\dagger_L(x)\partial_x\hat{\psi}_L(x) \right)$$

Giamarchi, *Quantum Physics in One Dimension* (2004)

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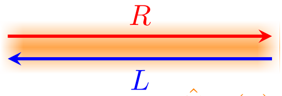


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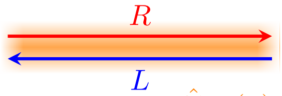


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Exact diagonalization in terms of **chiral boson fields**:

$$\hat{\mathcal{H}}_{LL}(x,t) = \frac{u}{2} \left\{ [\partial_x \hat{\phi}_+(x - ut)]^2 + [\partial_x \hat{\phi}_-(x + ut)]^2 \right\}$$

- Chiral boson fields are decoupled from each other.

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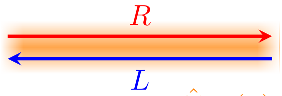


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$$K = \sqrt{\tfrac{2\pi v_{\mathrm{F}}+g_4-g_2}{2\pi v_{\mathrm{F}}+g_4+g_2}} \leq 1$$

- Chiral boson fields are decoupled from each other.
- Different interaction strength ($K$) correspond to different couple of chiral fields (Bogoliubov transformation).

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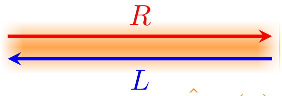


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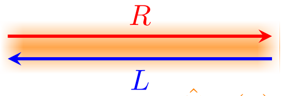


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- Chiral boson fields are decoupled from each other.
- Different interaction strength ($K$) correspond to different couple of chiral fields (Bogoliubov transformation).
- LLs have peculiar features: zero bias anomaly in DOS (power-law behavior), fractionalization phenomena, . . .

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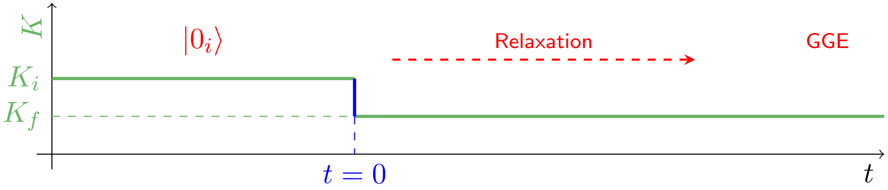
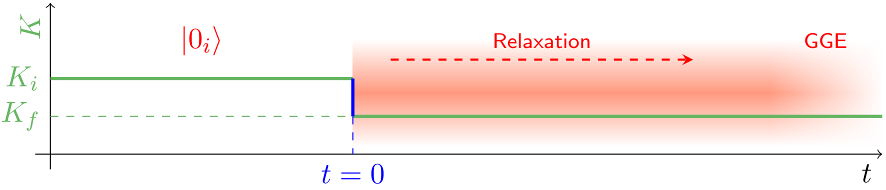
# SUDDEN INTERACTION QUENCH






Focusing on the evolution **after** the quench:

$$\hat{\mathcal{H}}_{LL,f}(x, t > 0) = \frac{u}{2} \left\{ [\partial_x \hat{\phi}_{f,+}(x - ut)]^2 + [\partial_x \hat{\phi}_{f,-}(x + ut)]^2 \right\}$$

$$|0_i\rangle \propto \exp\left[ \sigma[K_i, K_f] \int_{-\infty}^{+\infty} \left( \partial_x \hat{\phi}_{f,+}(x) \right) \hat{\phi}_{f,-}(x) \right] |0_f\rangle$$

Quench-induced entanglement between the two chiral channel $+$ and $-$.

# CROSS-CORRELATORS RELAXATION

Quench-induced entanglement ⟶ Non-vanishing local cross correlator between bosonic fields

$$D_{cc}(t,\tau) = \langle \hat{\phi}_{+,f}(t-\tau)\hat{\phi}_{-,f}(t)\rangle + \langle \hat{\phi}_{+,f}(t)\hat{\phi}_{-,f}(t-\tau)\rangle - \langle \hat{\phi}_{+,f}(t-\tau)\hat{\phi}_{-,f}(t-\tau)\rangle - \langle \hat{\phi}_{+,f}(t)\hat{\phi}_{-,f}(t)\rangle$$

Quench-induced entanglement $\longrightarrow$ Non-vanishing local cross correlator between bosonic fields

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In the long time limit $t \gg \tau$, the cross correlator relaxes to zero as a power-law

$$D_{cc}(t,\tau) \approx \frac{d(\tau)}{t^2} \to 0$$

- Universal: the exponent is independent on both $K_i$ and $K_f$
- Pure non-equilibrium effect
- Probe of quench-induced entanglement and relaxation dynamics

Local (lesser) non-equilibrium spectral function (NESF)

$$\mathcal{A}^{<}(\omega,t) = \frac{-i}{2\pi}\int e^{i\omega\tau}G^{<}(t,t-\tau)d\tau \qquad G^{<}(t,t-\tau) = i\langle\hat{\psi}^{\dagger}(t-\tau)\hat{\psi}(\tau)\rangle$$

$$\mathcal{A}^{<}(\omega,t) - \mathcal{A}^{<}_{\infty}(\omega) \approx \frac{U_{\mathcal{A}}(\omega)}{t^2} + \frac{M_{\mathcal{A}}(\omega,t)}{t^{\nu}}$$

with

$$M_{\mathcal{A}} \approx e^{i\omega t}$$

$$\nu(K_i,K_f) \gtrsim 1$$

- universal power-law induced by the peculiar $t$ dependence of $D_{cc}$
- non universal power-law (breaking of time translational invariance)

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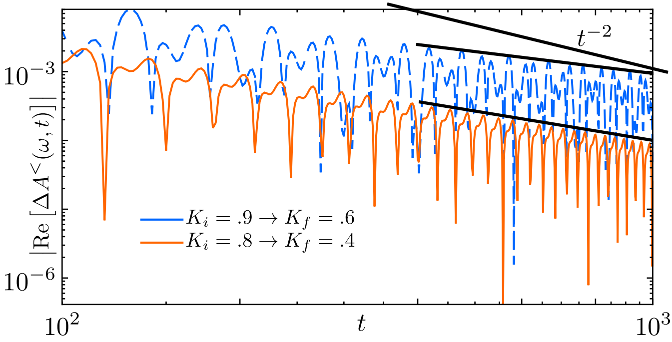
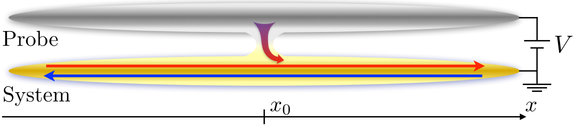
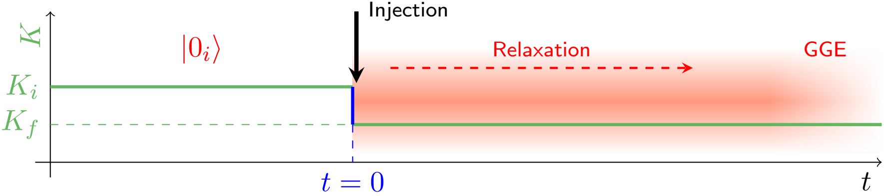



Although present, the $t^{-2}$ relaxation is subleading!

Injection from a biased probe, locally tunnel coupled with the quenched system




$$\hat{\mathcal{H}}_p(x) = i v_{\mathrm{F}} \hat{\chi}^\dagger(x) \partial_x \hat{\chi}(x) \qquad \hat{H}_{\mathrm{T}}(t) = \theta(t) \left[ \lambda \hat{\psi}_R^\dagger(x_0) \hat{\chi}(x_0) + \mathrm{h.c.} \right]$$




Breaking of inversion symmetry (injection of $R$ electrons): allows to study also fractionalization of the injected currents.

$$\hat{\psi}_R^\dagger \propto e^{i\sqrt{2\pi}(A_+ \hat{\phi}_+ + A_- \hat{\phi}_-)}$$

$$I_\eta(V,t) = \partial_t \int_{-\infty}^{\infty} \langle \delta n_\eta(V,x,t)\rangle \, dx$$

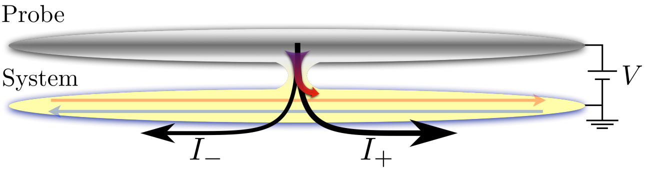
Variation of chiral ($\eta = \pm$) charge density (order $|\lambda|^2$)

$$= \left(\frac{1+\eta K_f}{2}\right)|\lambda|^2 \, 2\mathrm{Re}\left[\int_0^t iG^<(t, t-\tau) G_p^>(\tau)\sin(V\tau)d\tau\right]$$

$$I_\eta(V,t) - I_\eta^\infty(V) \approx \left(\frac{1+\eta K_f}{2}\right)\left(\frac{U_I(V)}{t^2} + \frac{M_I(V,t)}{t^{1+\nu}}\right)$$

$\nu(K_i, K_f) \gtrsim 1$

$M_I \approx \cos(Vt)$



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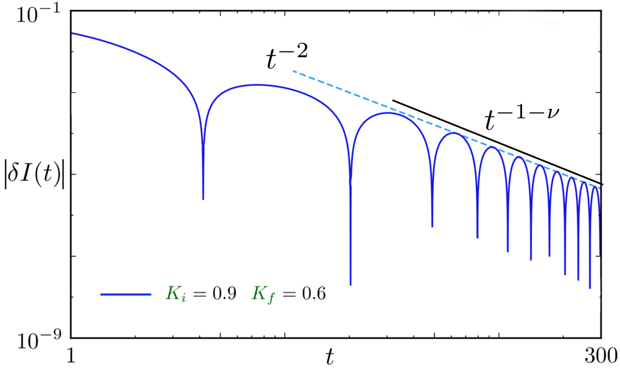


- The universal decay is leading!
- Competition is still strong!

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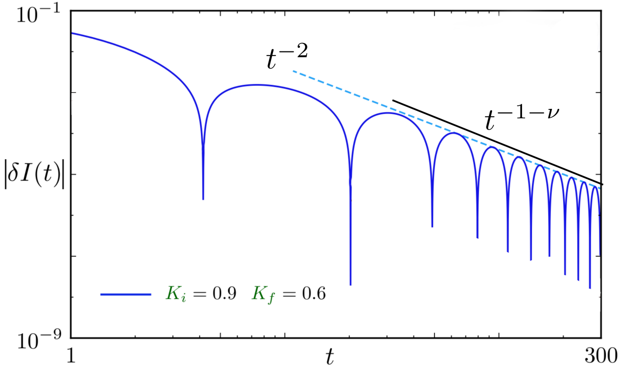



- The universal decay is leading!
- Competition is still strong!
- Charge fractionalization is "trivial"!

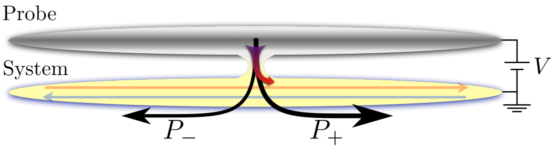
$$P_\eta(V,t) = \partial_t \int_{-\infty}^{\infty} \langle \delta\mathcal{H}_\eta(V,x,t)\rangle dx$$

Variation of chiral ($\eta = \pm$) hamiltonian density (order $|\lambda|^2$)

$$= 2|\lambda|^2 \mathrm{Im}\left[\int_0^t G_p^>(\tau)(\partial_{\bar{t}} - \eta u_f \partial_\xi) G_R^<(\xi;\bar{t},t-\tau)\Big|_{\substack{\xi=0\\ \bar{t}=t}} \cos(V\tau)d\tau\right]$$

$$P_\eta(V,t) - P_\eta^\infty(V) \approx \mathcal{R}_\eta(K_f)\left(\frac{U_P^A(V)}{t^2} + \frac{M_P^A(V,t)}{t^{2+\nu}}\right) + \xi(K_i,K_f)\left(\frac{U_P^B(V)}{t^2} + \frac{M_P^B(V,t)}{t^{2+\nu}}\right)$$

$$\nu(K_i,K_f) \gtrsim 1$$
$$M_P \approx \cos(Vt)$$



# ENERGY CURRENT

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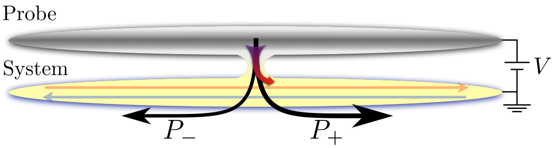
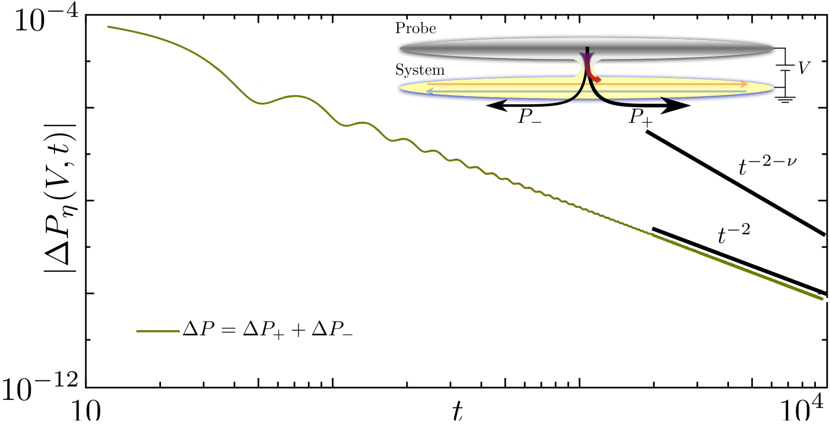
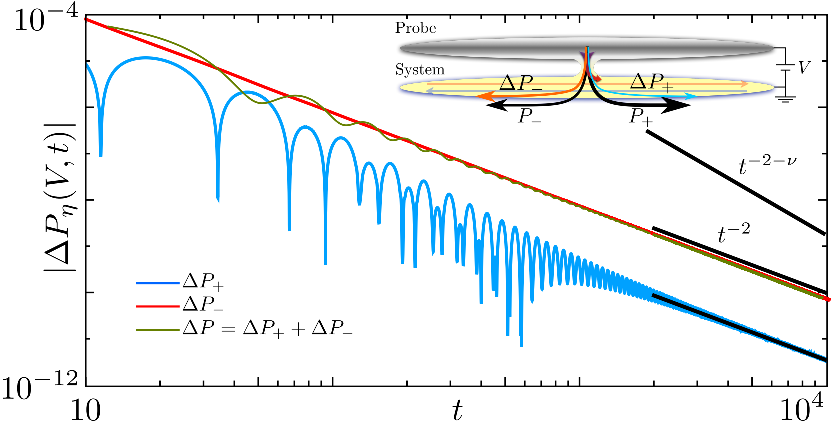





$$K_i = 0.9 \quad \rightarrow \quad K_f = 0.6$$

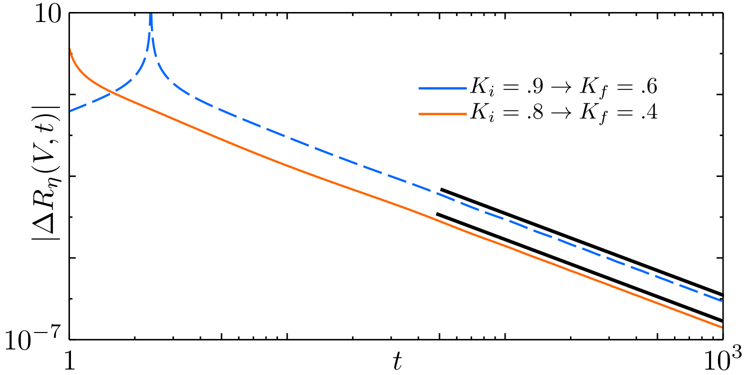



$K_i = 0.9 \quad \rightarrow \quad K_f = 0.6$

The left moving component of the transient ($\Delta P_-$) displays an almost perfect $t^{-2}$ behavior, even after a short time.

Fractionalization ratio

$$R_+(V,t) = \frac{P_+(V,t)}{P_+(V,t) + P_-(V,t)} \qquad \Delta R_+(V,t) = R_+(V,t) - R_+^\infty(t)$$



# CONCLUSIONS

Interaction quench
in Luttinger liquid

Non-equilibrium
entangled state

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Transport properties

Interaction quench in Luttinger liquid → Non-equilibrium entangled state

Non-equilibrium entangled state → Universal relaxation $\propto t^{-2}$ of local cross-correlators

Universal relaxation $\propto t^{-2}$ of local cross-correlators → Subleading in local NESF

Subleading in local NESF → Transport properties

Transport properties → Charge current:

- $t^{-2}$ is leading
- "trivial" fractionalization

Interaction quench in Luttinger liquid

Non-equilibrium entangled state

Universal relaxation $\propto t^{-2}$ of local cross-correlators

Subleading in local NESF

Transport properties

**Energy current:**

- $t^{-2}$ emerges even more clearly
- non-trivial partitioning:
  - Predominance of the "$-$" channel
  - Direct fingerprint of quench-induced dynamics

**Charge current:**

- $t^{-2}$ is leading
- "trivial" fractionalization

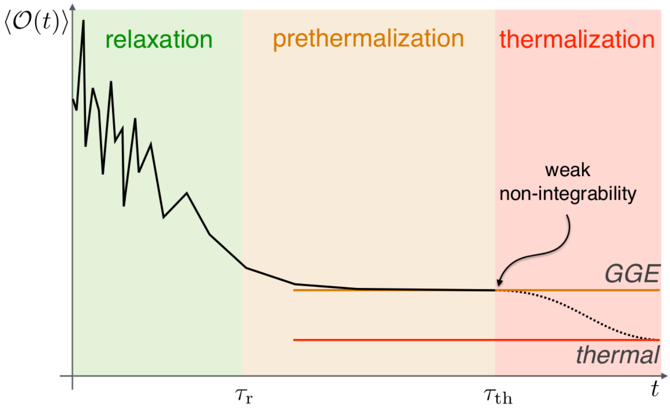
**A. Calzona, F. M. Gambetta, F. Cavaliere, M. Carrega, and M. Sassetti**

**arXiv:1706.01676**

# Thank you for your attention!

$$\nu = \frac{K_f^4 + K_i^2 + 3K_f^2(1 + K_i^2)}{8K_f^2 K_i}$$

Integrable system $\rightarrow$ GGE
Non-integrable system (ETH) $\rightarrow$ Thermalization




Polkovinkov, RMP (2012)
D'Alessio, Adv. Phys (2016)
Rigol, PRL (2007)

# FRACTIONALIZATION

- Breakdown of Fermi liquid model, **Luttinger liquid** instead!
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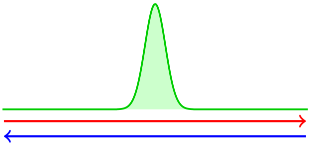

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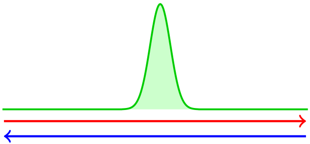

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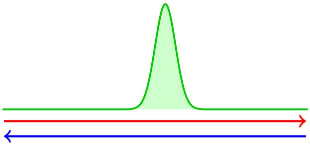

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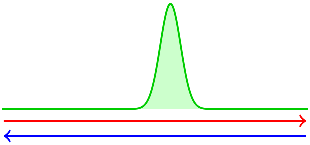

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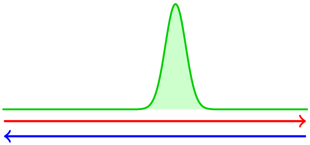

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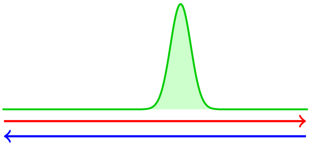

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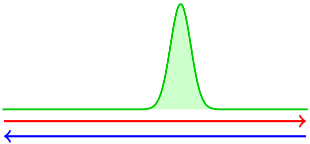

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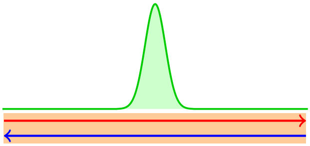
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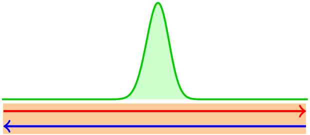

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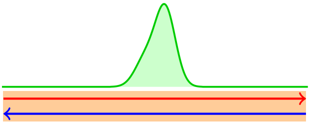

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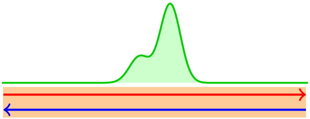

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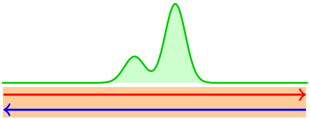

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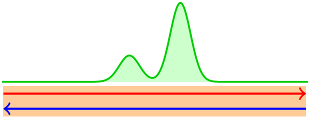

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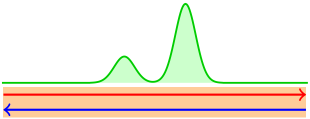

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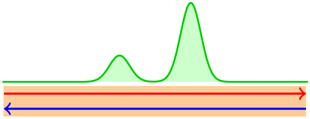

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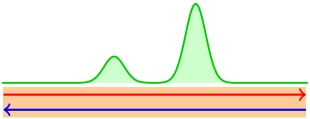

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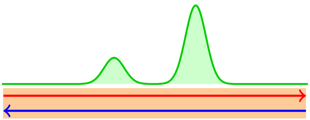

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