Optimal quantum interference thermo-electric heat engine with edge states

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Mesoscopic Transport and Quantum Coherence
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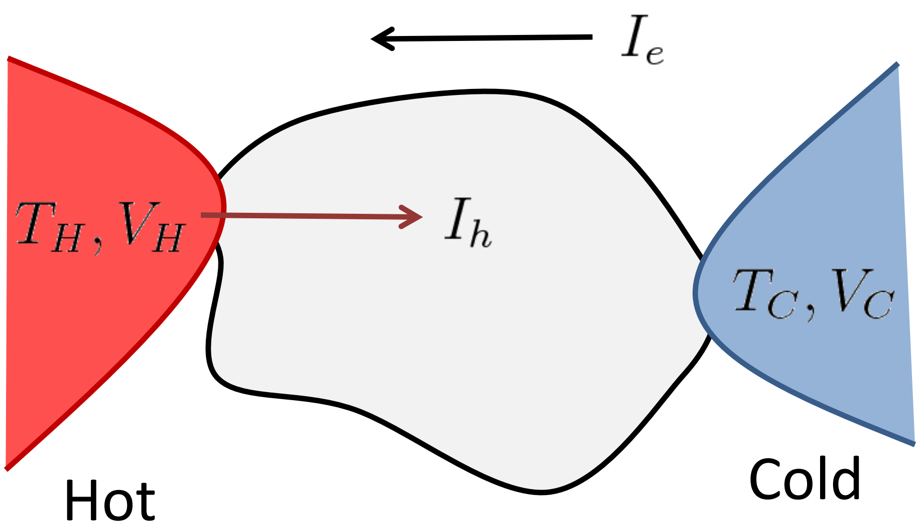
Mesoscopic thermoelectric heat engine

Steady state heat to electrical work conversion in mesoscopic conductors



Performance

- Power

$$P = I_e V$$

Electrical current I_e flowing against applied bias

$$V = V_H - V_C > 0$$

- Efficiency

$$\eta = \frac{P}{I_h} \leq \eta_C$$

bounded by Carnot η_C .

Motivation

- Proof-of-principle for nanoscale waste heat recovery.
- Role of coherence in heat engine performance.
- Investigate fundamentals of energy transport.

Optimal performance

Two-terminal heat engine, linear response

Transmission

- Energy dependence of

$$\mathcal{T}(\varepsilon)$$

governs thermoelectric properties

- Maximizing output power

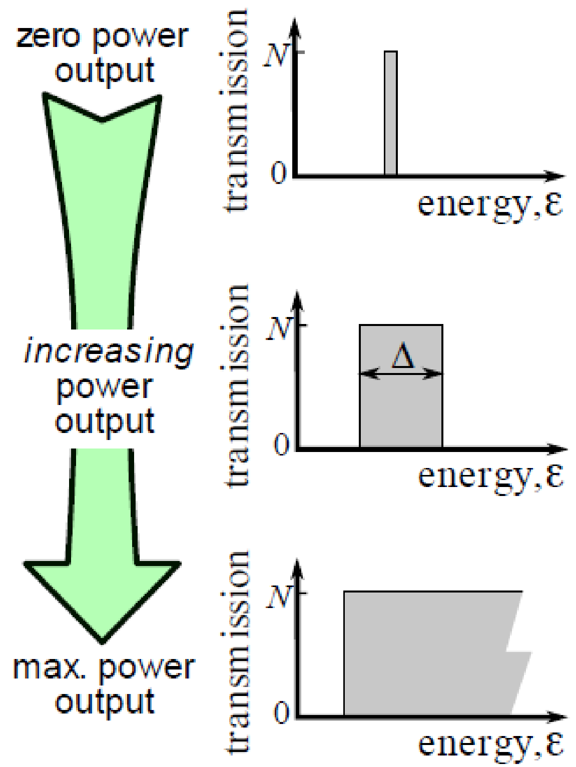
$$\Longrightarrow$$

Step function in energy

Q1: Can a purely interference based TE-heat engine be optimal?

Optimization

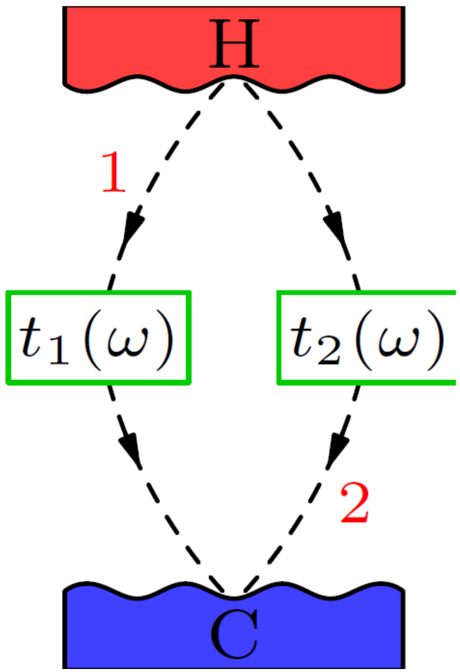
Transmission giving optimal η for given P.



Whitney, PRL 2014

Double slit interferometer

Generic interferometer



Scattering amplitude

$$A(\omega) \propto t_1(\omega) + e^{i\phi_0} t_2(\omega)$$

ϕ_0 Phase due to Aharanov-Bohm, ..

No individual TE-effect

$$T_i = |t_i(\omega)|^2$$

Energy independent

$$\Longrightarrow \quad t_i(\omega) = \sqrt{T_i} e^{i\alpha_i(\omega)}$$

Energy dependence

Scattering probability

$$|A(\omega)|^2 \propto T_1 + T_2 + 2\sqrt{T_1 T_2} \cos[\alpha_1(\omega) - \alpha_2(\omega) - \phi_0]$$

Double slit interferometer

$$|A(\omega)|^2 \propto T_1 + T_2 + 2\sqrt{T_1 T_2}\cos[\alpha_1(\omega) - \alpha_2(\omega) - \phi_0]$$

Conditions for transmission step

- Symmetric interferometer, $T_1 = T_2$.
- Sharp jump $\pi \longrightarrow 0$ for $\alpha_1(\omega) - \alpha_2(\omega) - \phi_0$ at $\omega = \omega_0$.

$\Longrightarrow$

$$\mathcal{T}(\omega) = \theta(\omega - \omega_0)$$

Step function transmission

Q1: Can a purely interference based TE-heat engine be optimal?

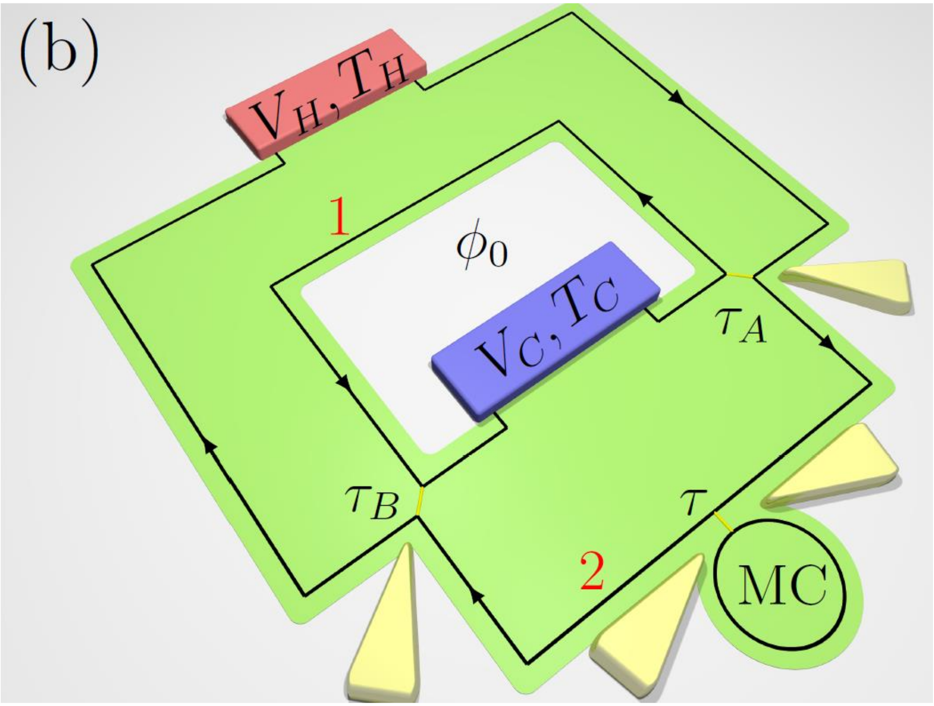
A1: Yes

$\Longrightarrow$

Q2: Is there a possible experimental realization of such optimal engine?

Edge state Mach-Zehnder with capacitor

Extending Hofer, Sothmann, PRB 2015



Properties

- Quantum point contacts

  τ_A, τ_B

- Mesoscopic capacitor (MC)

  τ, Δ, ω_0 Fève et al, Science 2007

- Contacts

  $T_C, T_H \quad V_C, V_H$

- Interferometer

  ϕ_0

Total transmission amplitude (equal arm lengths)

$$t(\omega) = \sqrt{\tau_A \tau_B} - \sqrt{(1-\tau_A)(1-\tau_B)} e^{i[\alpha(\omega)+\phi_0]}$$

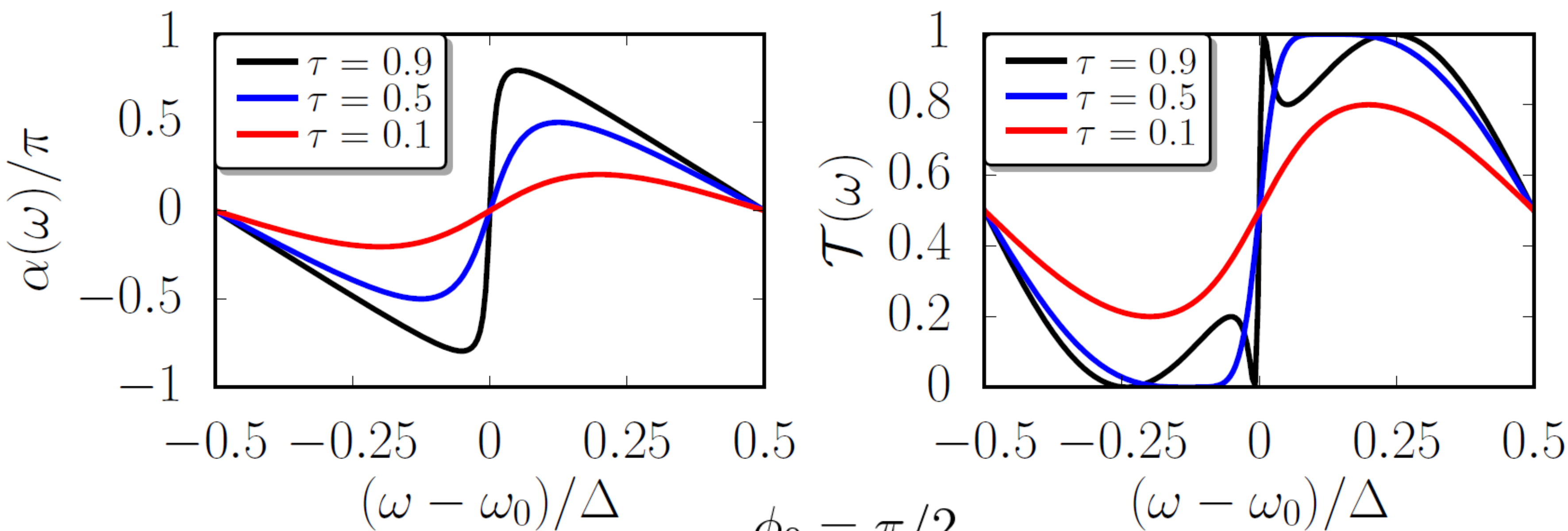
$$\alpha(\omega) = 2 \arctan \frac{\sqrt{\tau} \sin\left(2\pi \frac{\omega-\omega_0}{\Delta}\right)}{1 - \sqrt{\tau} \cos\left(2\pi \frac{\omega-\omega_0}{\Delta}\right)}$$

Scattering phase at MC

Transmission properties

Semitransparent splitters $\tau_A = \tau_B = 1/2$ (symmetry condition),

$$\mathcal{T}(\omega) = \frac{\left[\sin\left(\frac{\phi_0}{2}\right) - \sqrt{\tau}\sin\left(\frac{\phi_0}{2} - 2\pi\frac{\omega-\omega_0}{\Delta}\right)\right]^2}{1 - 2\sqrt{\tau}\cos\left(2\pi\frac{\omega-\omega_0}{\Delta}\right) + \tau}.$$

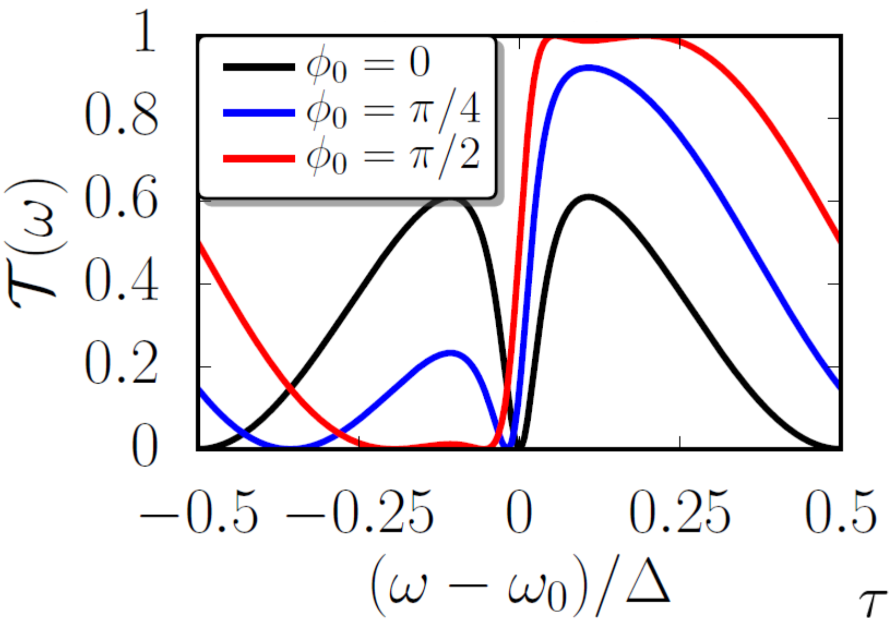


$\phi_0 = \pi/2$

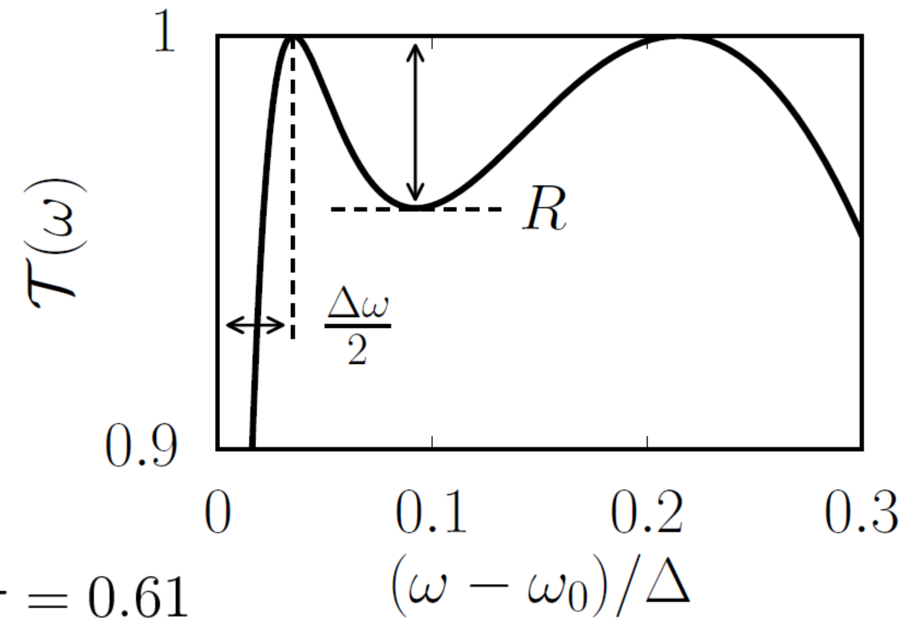
Effective phase shift

$$\Delta\phi \equiv \max_{\omega}\{\alpha(\omega)\} - \min_{\omega}\{\alpha(\omega)\} \Longrightarrow \Delta\phi = \pi \text{ for } \tau = 1/2$$

Transmission properties



$\tau = 0.61$



- Phase dependent symmetries

$$\phi_0 = \pi/2 \qquad \mathcal{T}(\omega - \omega_0) = 1 - \mathcal{T}(-[\omega - \omega_0])$$

$$\phi_0 = 0 \qquad \mathcal{T}(\omega - \omega_0) = \mathcal{T}(-[\omega - \omega_0])$$

- Filter analogy, $\tau > 0.5$, $\phi_0 = \pi/2$

$$R = 1/2 - \sqrt{\tau(1-\tau)}$$ Ripple

$$\Delta\omega = \frac{\Delta}{\pi}\left[\arcsin\left(\frac{1}{\sqrt{2\tau}}\right) - \frac{\pi}{4}\right]$$ Transition width, roll-off

Thermoelectric scattering theory

Linear response theory (non-interacting), charge and heat currents

Butcher, 1990

$$\begin{pmatrix} I_e \\ I_h \end{pmatrix} = \begin{pmatrix} \mathcal{L}_{eV} & \mathcal{L}_{eT} \\ \mathcal{L}_{hV} & \mathcal{L}_{hT} \end{pmatrix} \begin{pmatrix} F_V \\ F_T \end{pmatrix}$$

Thermodynamic forces

$$F_V = eV/k_B T \qquad F_T = \Delta T/(k_B T)^2$$

where $V = V_H - V_C, \quad \Delta T = T_H - T_C$

Onsager matrix

$$\begin{pmatrix} \mathcal{L}_{eV} & \mathcal{L}_{eT} \\ \mathcal{L}_{hV} & \mathcal{L}_{hT} \end{pmatrix} = \frac{1}{h} \int d\omega\, \mathcal{T}(\omega) \xi(\omega) \begin{pmatrix} e & e\omega \\ \omega & \omega^2 \end{pmatrix}$$

where $\xi(\omega) = \left(2\cosh\frac{\omega}{2}\right)^{-2}$

Power and efficiency

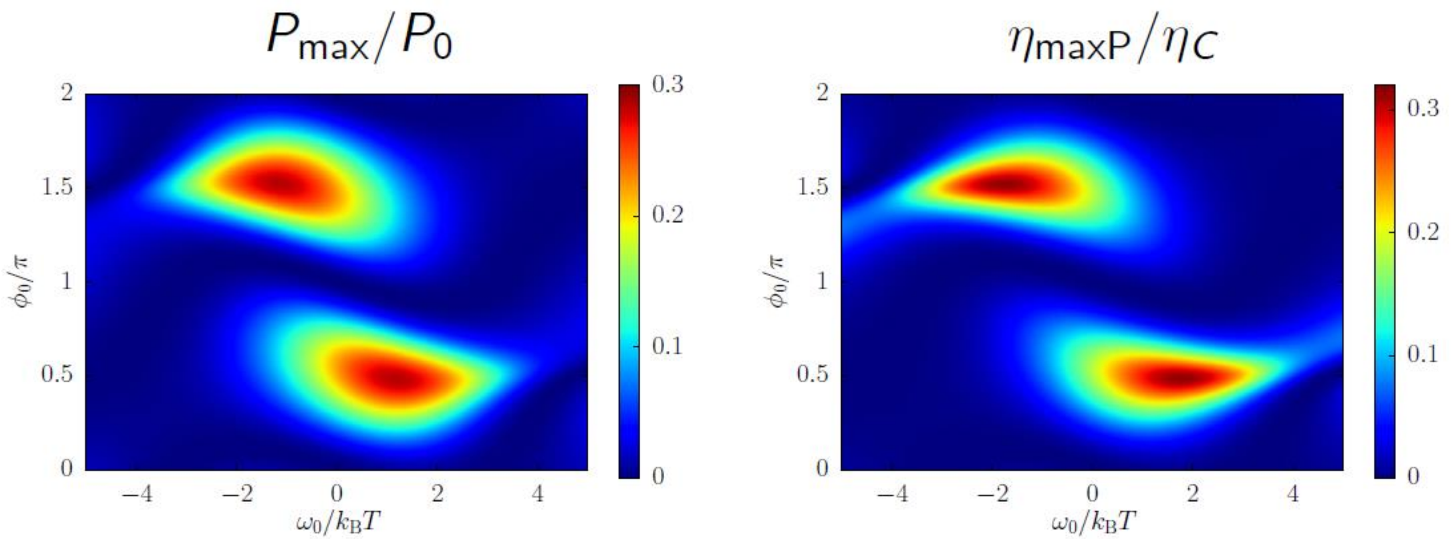
Maximum power generated, with respect to voltage

$$P_{\max} = \frac{k_B T}{4e} \frac{(\mathcal{L}_{eT})^2}{\mathcal{L}_{eV}} (F_T)^2.$$

Efficiency at maximum power

$$\eta_{\mathrm{maxP}} = \frac{P_{\max}}{I_h} = \frac{\eta_C}{2e} \frac{(\mathcal{L}_{eT})^2}{2\mathcal{L}_{eV}\mathcal{L}_{hT} - \mathcal{L}_{eT}}$$

$$P_0 = \frac{(k_B \Delta T)^2}{h}$$



Close-to-optimal performance

Optimal, single mode performance (step function transmission)

$$\mathcal{T}(\omega) = \theta(\omega - \omega_0) \Rightarrow$$

$$P_{\max} = 0.32 \frac{(k_B \Delta T)^2}{h} \quad \text{for} \quad \omega_0 = 1.16 k_{\mathrm{B}} T$$

$$\eta_{\mathrm{maxP}} = 0.35 \eta_C$$

Numerical optimization

Optimizing $\mathcal{T}(\omega)$ over $0 \leq \tau \leq 1, 0 \leq \phi_0 \leq \pi, \omega_0, \Delta$

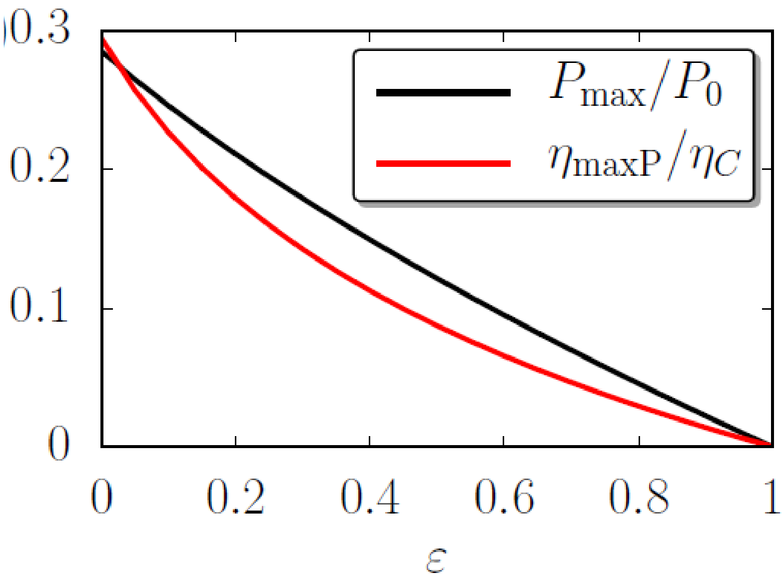
- Parameters $\tau = 0.61, \phi_0 = 0.52\pi, \omega_0 = 1.17 k_B T, \Delta = 24.2 k_B T$
- Values

$$P_{\max} = 0.285 \frac{(k_B \Delta T)^2}{h} \qquad \eta_{\mathrm{maxP}} = 0.29 \eta_C$$

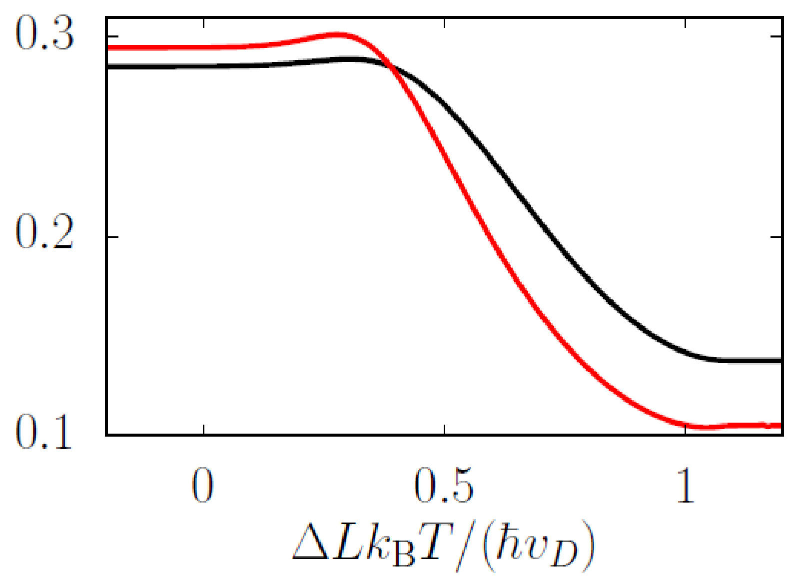
90% ⟵ of optimal ⟶ 83%

Show-stoppers?

Dephasing



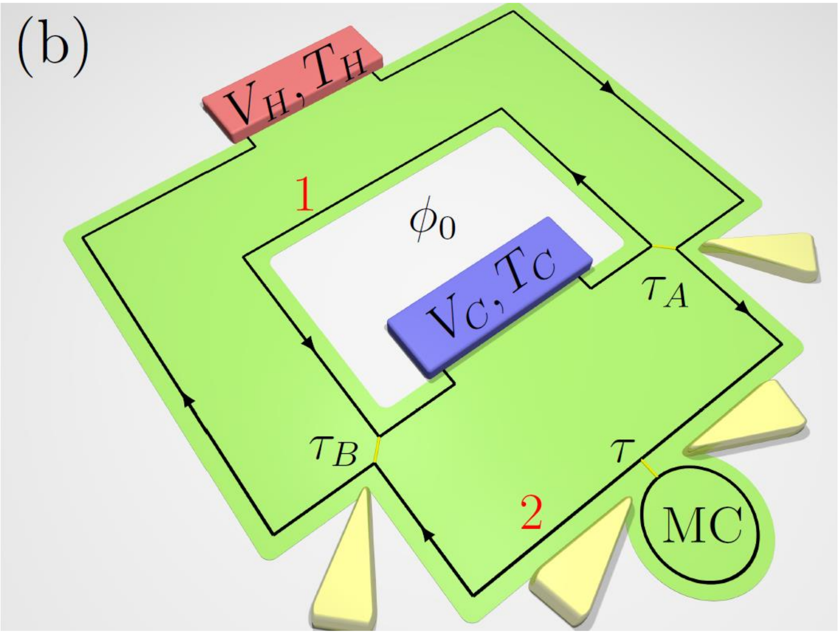
Asymmetry



- Survives for moderate dephasing strength $0 \le \epsilon \le 1$
- Zero for complete dephasing.
- Not very sensitive to arm length asymmetry $\Delta L = L_1 - L_2$

Conclusions

- Interference-only thermoelectrics potentially optimal
- Close-to-optimal performance in edge-state setup
- Not very sensitive to dephasing and asymmetry



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